Electronic Noise

- Dynamic range in the analog domain
  - Resistor noise
  - Amplifier noise
  - Maximum signal levels
- Tow-Thomas Biquad noise example
- Implications on power dissipation

Analog Dynamic Range

- Finite precision effects in digital filters are rapidly becoming negligible
  - Floating point digital filters with huge mantissas will be reduced to negligible cost
  - The only fixed-point numbers will come from ADCs
- But we will <u>always</u> have thermal noise

Analog Dynamic Range

- Let's say you've selected the poles and zeroes of your analog filter transfer function
- Of the infinitely many ways to build a filter with a given transfer function, **each of those ways has a different output noise!**

Analog Dynamic Range

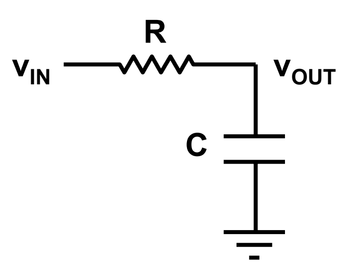
- The job of a high-performance analog filter designer is to get reasonably close to the optimal noise for a given transfer function
  - Not the absolute minimum noise, just close
- The job of a mixed-signal chip architect is to appreciate filter noise and to be able to model filters well enough to know that a given dynamic range objective is feasible

Analog Dynamic Range

- We'll begin our adventure in analog filter implementation by looking at the noise in resistors and simple RC filters…

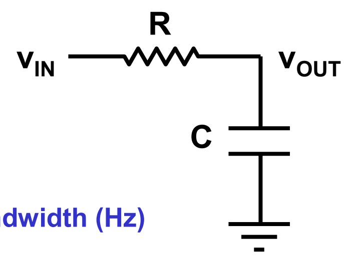
Resistor Noise

- Capacitors are noiseless
- Resistors have thermal noise
  - This noise is uniformly distributed from dc to infinity
  - Frequency-independent noise is called "white noise"



Resistor Noise

- Resistor noise has
  - A mean value of zero
  - A mean-squared value



$$\overline{v_n^2} = 4k_B T_r R \Delta f$$

ohms

Volts²

measurement bandwidth (Hz)

absolute temperature (°K)

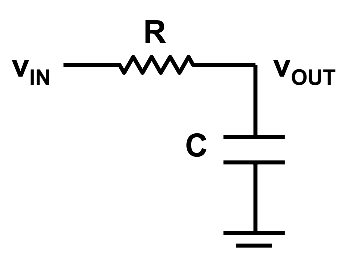
Boltzmann's constant = 1.38e-23 J/°K

Resistor Noise

- Resistor rms noise voltage in a 10Hz band centered at 1kHz is the same as resistor rms noise in a 10Hz band centered at 1GHz

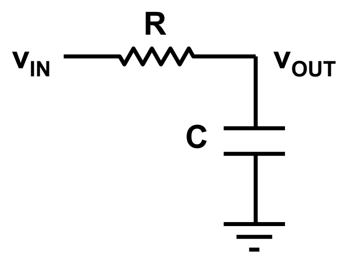
- Resistor noise spectral density, N_0, is the rms noise per √Hz of bandwidth:

$$N_0 = \sqrt{\frac{\overline{v_n^2}}{\Delta f}} = \sqrt{4k_B T_r R}$$



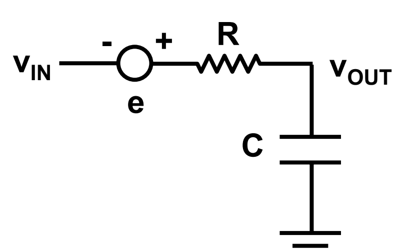
Resistor Noise

- Don't bother to remember Boltzmann's constant
- Instead, remember forever that N_0 for a **1kΩ** resistor at room temperature is **4nV/√Hz**
- Scaling R,
  - A 10MΩ resistor gives 400nV/√Hz
  - A 50Ω resistor gives 0.9nV/√Hz
- Or, remember that $k_B T_r = 4 \times 10^{-21}$ J ($T_r = 17\ ^oC$)



Resistor Noise

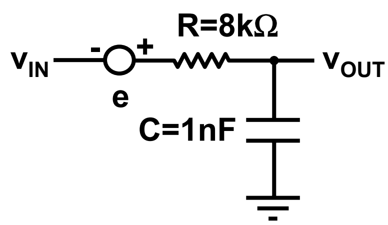
- Resistor noise gives our filter a non-zero output when $v_{IN}=0$
- In this simple example, both the input signal and the resistor noise obviously have the same transfer functions to the output
- Since noise has random phase, we can use any polarity convention for a noise source (but we have to use it consistently)



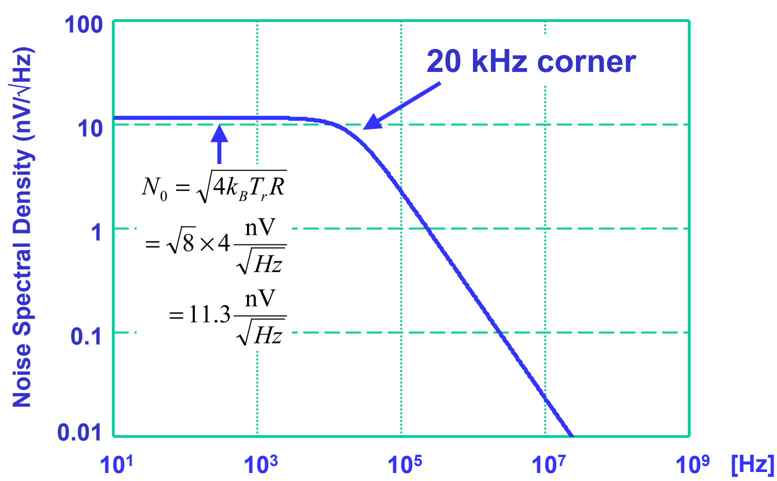
Resistor Noise

- What is the thermal noise of the RC filter?
- Let's ask SPICE!
  Netlist:

```
Noise from RC LPF
vin vin 0 ac 1V
r1 vin vout 8kOhm
c1 vout 0 1nF
.ac dec 100 10Hz 1GHz
.noise V(vout) vin
.end
```



LPF1 Output Noise Density



$$N_0 = \sqrt{4k_B T_r R} = \sqrt{8} \times 4 \frac{\text{nV}}{\sqrt{Hz}} = 11.3 \frac{\text{nV}}{\sqrt{Hz}}$$

Total Noise

- Suppose we want to know the value of v_o "now", what's the standard deviation error? (E.g. on the display of a volt-meter connected to v_o).
- Answer:

$$\overline{v_o^2} = \int_0^\infty 4k_B TR \left| H(2\pi j f) \right|^2 df$$

Total Noise

- Note that noise is integrated in the mean-squared domain, because noise in a bandwidth df around frequency f_1 is uncorrelated with noise in a bandwidth df around frequency f_2
  - Powers of uncorrelated random variables add
  - Squared transfer functions appear in the mean-squared integral

Total Noise

$$\overline{v_o^2} = \int_0^\infty 4k_B TR \left| H(2\pi jf) \right|^2 df$$

$$= \int_0^\infty 4k_B TR \left| \frac{1}{1 + 2\pi jfRC} \right|^2 df$$

$$= \frac{k_B T}{C}$$

Total Noise

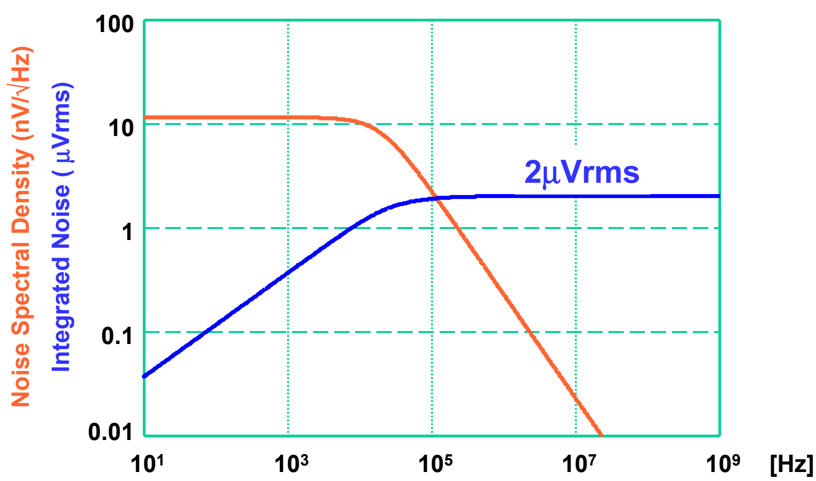
$$\overline{v_o^2} = \frac{k_B T}{C}$$

- This interesting and somewhat counterintuitive result means that even though resistors provide the noise sources, capacitors set the total noise
- For a given capacitance, as resistance goes up, the increase in noise density is balanced by a decrease in noise bandwidth

kT/C Noise

- The rms noise voltage of the simplest possible (first order) filter is $\sqrt{k_B T/C}$
- For 1pF, $\sqrt{k_B T/C}$ = 64 μV-rms (at 298°K)
- 1000pF gives 2 μV-rms
- The noise of a more complex filter is $\sqrt{K \times k_B T/C}$
  K depends on implementation and features such as filter order

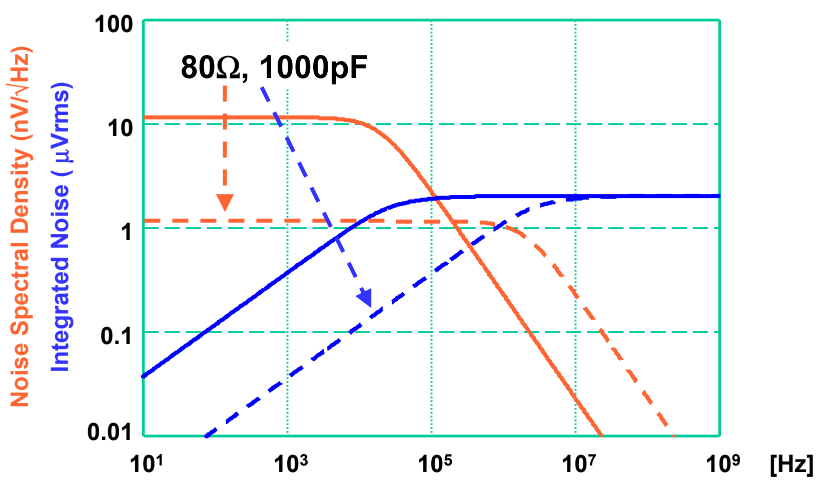
LPF1 Output Noise



LPF1 Output Noise

- Note that the integrated noise essentially stops growing above 100kHz for this 20kHz lowpass filter
- Beware of faulty intuition which might tempt you to believe that an 80Ω, 1000pF filter has lower integrated noise than our 8000Ω, 1000pF filter…

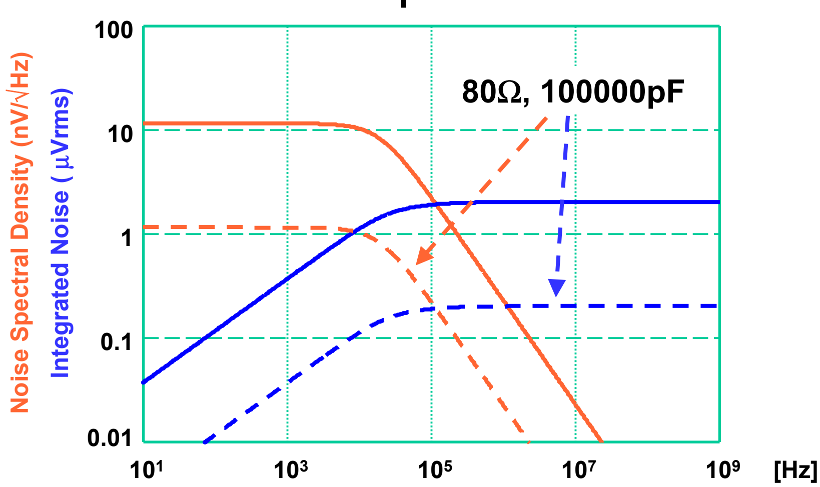
LPF1 Output Noise



LPF1 Output Noise

- Of course, an 80Ω, 100,000pF filter has both the same bandwidth AND lower integrated noise than our 8000Ω, 1000pF filter
- In the analog filter dynamic range game, the highest capacitance wins

LPF1 Output Noise



Analog Circuit Dynamic Range

- The biggest signal we can ever expect at the output of a circuit is limited by the supply voltage, V_{DD} hence (for sinusoids)

$$V_{\max}(rms) = \frac{1}{\sqrt{2}}\frac{V_{DD}}{2}$$

- The noise is

$$V_n(rms) = \sqrt{K\frac{k_B T}{C}}$$

- So the dynamic range in dB is:

$$DR = \frac{V_{\max}(rms)}{V_n(rms)} = \frac{V_{DD}\sqrt{C}}{\sqrt{8Kk_BT}} \qquad [V/V]$$

$$= 20\log_{10}\left(V_{DD}\sqrt{\frac{C}{K}}\right) + 75 \qquad [dB] \text{ with C in [pF]}$$

Analog Circuit Dynamic Range

- For integrated circuits built in modern CMOS processes, V_{DD} < 3V and C < 1nF (K = 1)
  – DR < 110dB
- For PC board circuits built with “old-fashioned” 30V opamps and discrete capacitors of < 100nF
  – DR < 140dB
  – A 30dB advantage!

Dynamic Range versus Bits

- Bits and dB are related:

$$DR = 2 + 6N \quad [\text{dB}]$$

  – see "quantization noise", later in the course

- Hence

110 dB	→	18 Bits
140 dB	→	23 Bits

Dynamic Range versus Power

- Each extra bit corresponds to 6dB
- 6dB means cutting noise power by 4!
- This translates into 4x larger capacitors
- To drive these at the same speed, G_m must increase 4x
- Power is proportional to G_m (for fixed supply and V_{dsat})

In analog circuits that are limited by thermal noise,
1 extra bit costs 4x power
E.g. 16Bit ADC at 200mW → 17Bit ADC at 800mW

Do not overdesign the dynamic range of analog circuits!

P.S. What is the cost of an extra bit in a 64Bit adder?

Active Filter Example

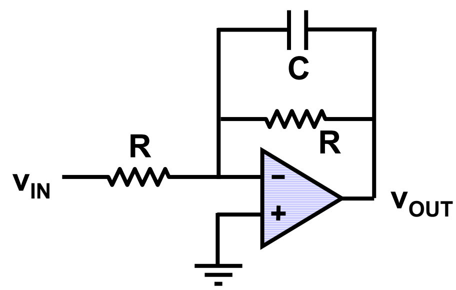
Frequency response:

$$H(s) = -\frac{1}{1 + sRC}$$

Total noise (see EE240):

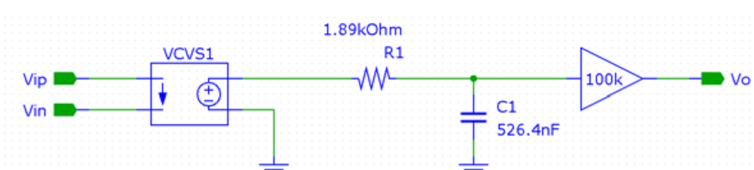
$$\sqrt{\overline{v_o^2}} = \sqrt{2\frac{k_B T}{C}}$$

$$K = 2$$



- Noise depends on filter topology
- Opamps contribute yet more noise …

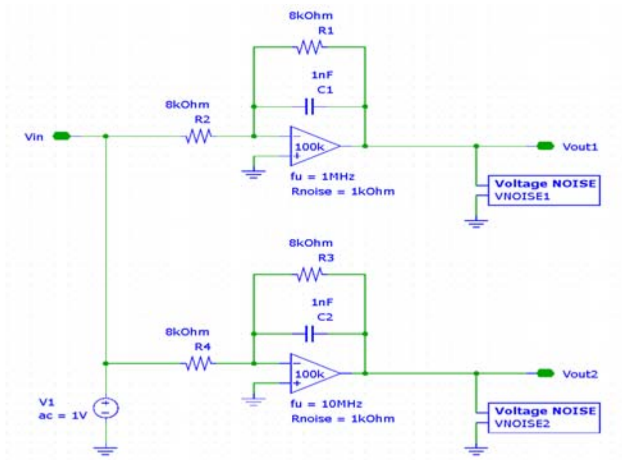
Behavioral Opamp Model



Specification	**Example**
Gain G	100k
Unity-gain bandwidth f_u	100 MHz
Input ref'd thermal noise	5 nV/√Hz

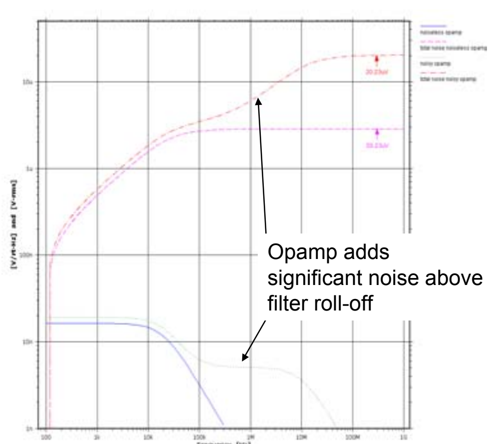
Beware of flicker noise and input current noise (BJTs).

SPICE Analysis



Noise Analysis

Opamp noise dominates in this example

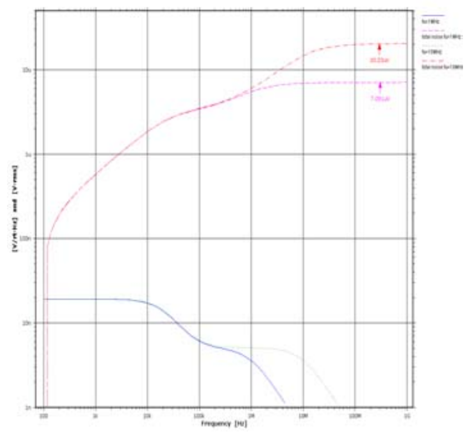


Opamp Bandwidth

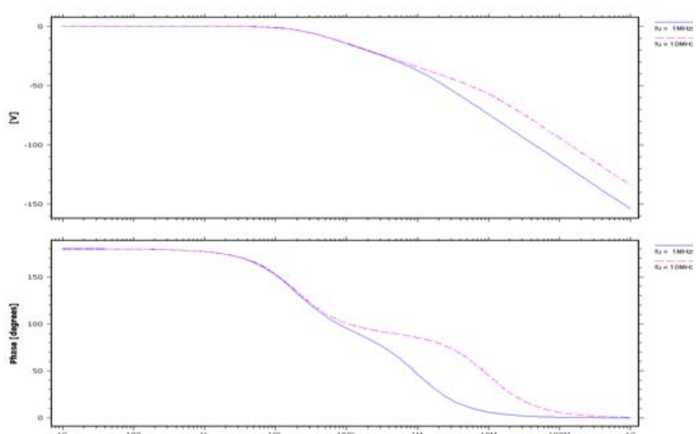
Minimize opamp bandwidth:

- f_u = 1MHz → 7µV-rms
- f_u = 10MHz → 20µV-rms

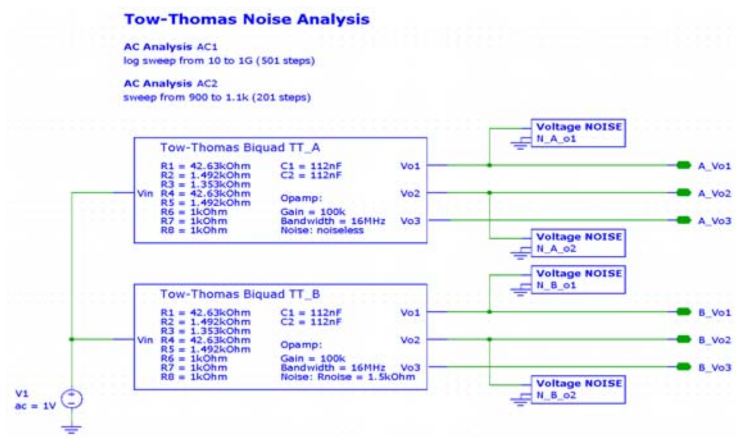
Of course, the opamp has to be fast enough to faithfully realize the 20kHz corner!



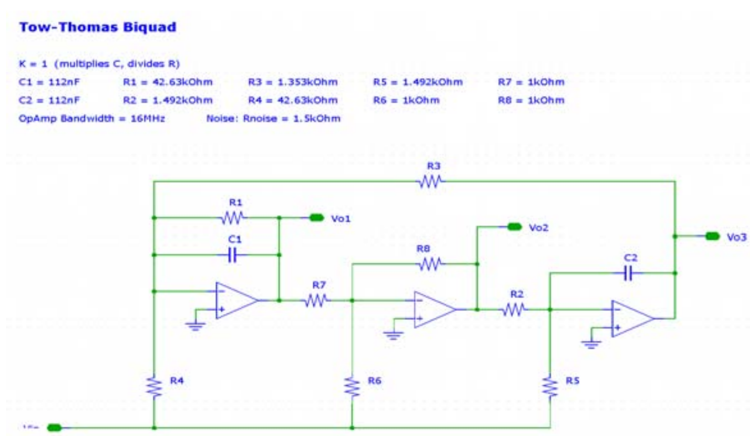
Frequency Response



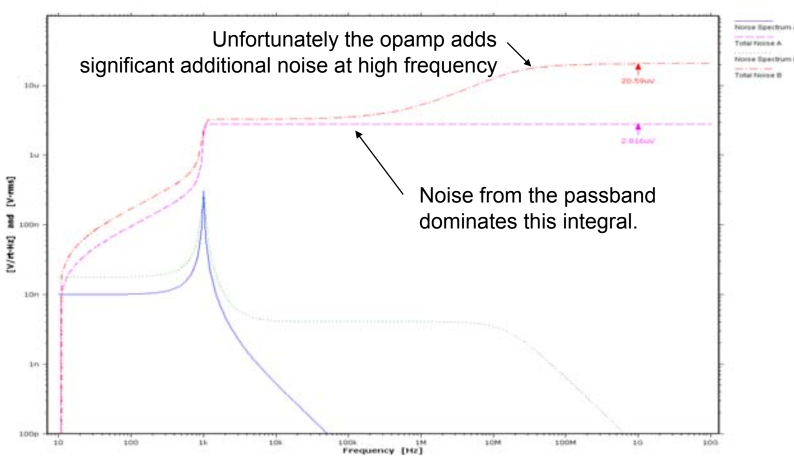
Tow-Thomas Noise Analysis



Tow-Thomas Biquad

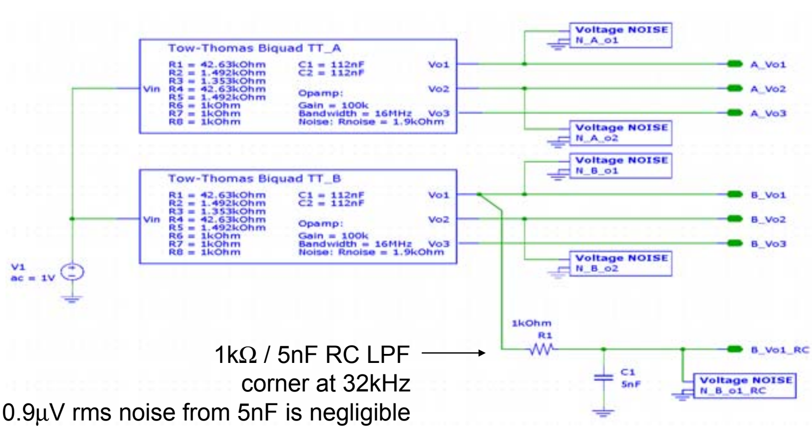


Bandpass Noise

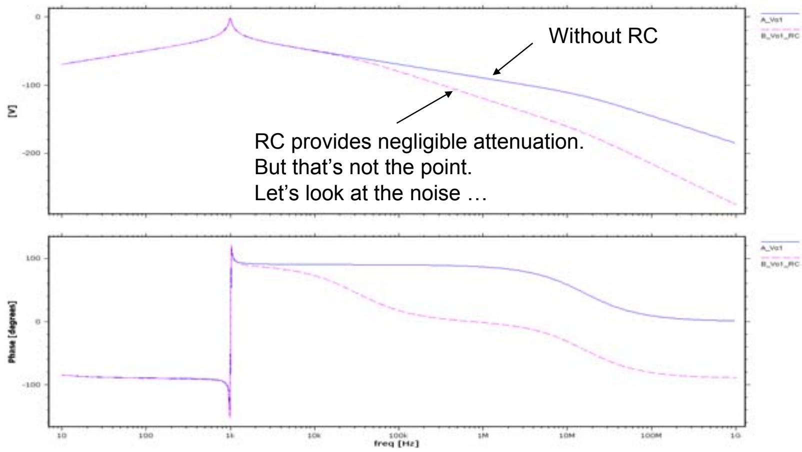


RC Filter Reduces BP Noise

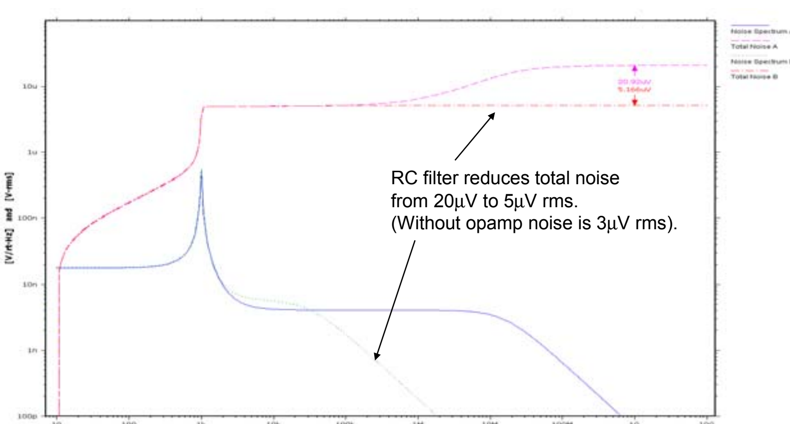
We cannot reduce the opamp noise or bandwidth … let's filter its noise!



BP Response with RC Filter



BP Noise after RC Filter

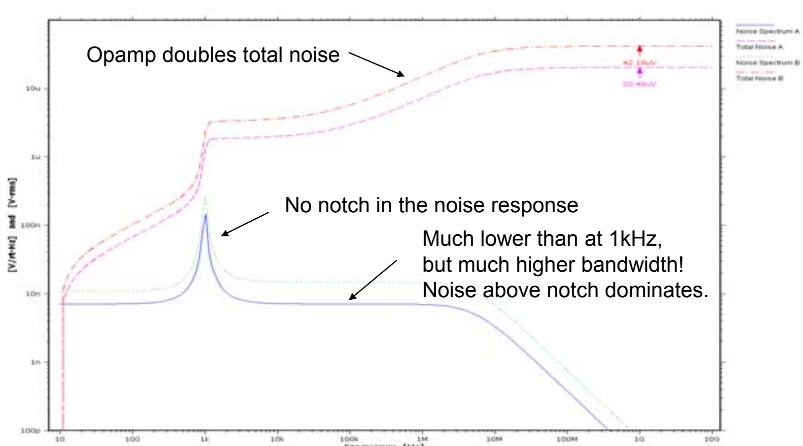


BP Dynamic Range

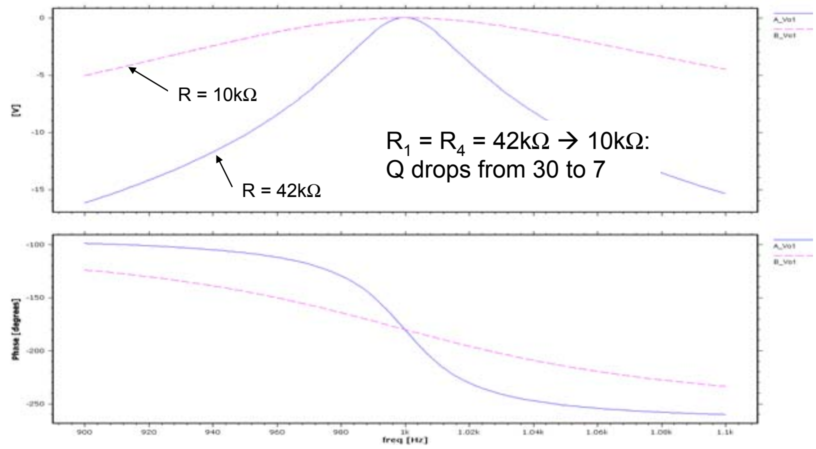
- Maximum sinewave input: 7.8V rms (limited by opamp)
- Noise: 5.2µV rms (with RC)
- Dynamic range: 123dB

→ No IC with integrated capacitors can get close to this dynamic range

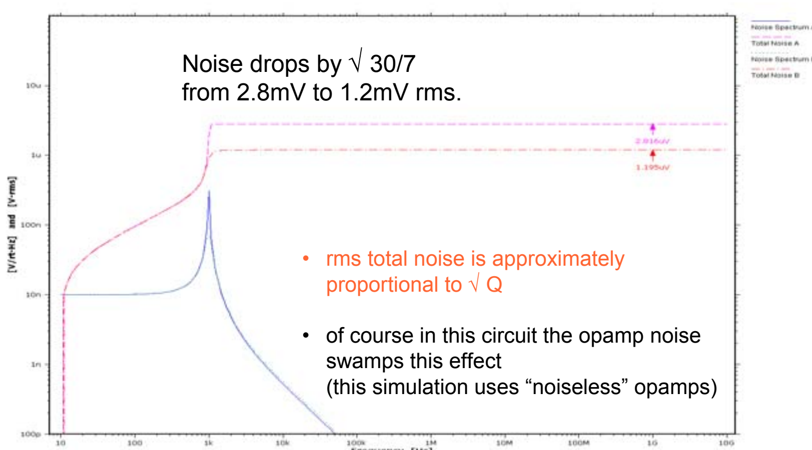
Bandstop Noise



Noise versus Pole Q



Noise versus Pole Q



- rms total noise is approximately proportional to √ Q
- of course in this circuit the opamp noise swamps this effect (this simulation uses “noiseless” opamps)

Noise Summary

- Thermal noise is a fundamental property of (electronic) circuits
- Noise is closely related to
  - Capacitor size and
  - Power dissipation
- In filters, noise is proportional to order, Q, and depends on implementation
- Operational amplifiers can contribute significantly to overall filter noise

