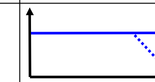
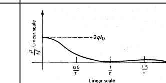
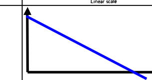
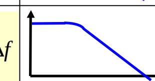
Lecture 3a.

Noise Tutorial

Gray, P.R. & Meyer. R.G.,
Analysis and Design of Analog Integrated Circuits.
(3rd Edition), Wiley (1992)

Noise Sources (resume)

Noise	Origin	Expression	Spectral density
thermal noise (Gaussian)	random fluctuations of velocity	$i_t^2 = 4kT\Delta f / R$	
shot noise (Gaussian)	due to DC current through p-n junction	$\overline{i_s^2} = 2qI_D \cdot \Delta f$	
flicker noise	traps in crystal lattice (semiconductors, carbon , etc.)	$\overline{i_f^2} = K_1 \frac{I_{DC}^{\alpha}}{f^{\beta}} \cdot \Delta f$	
burst noise	heavy-metal ions contamination. (gold , etc.)	$\overline{i_b^2} = K_2 \frac{I^c}{1+(f/f_c)^2} \cdot \Delta f$	
avalanche noise	due to avalanche breakdown in zener diodes		

Exam Questions

a) Noise Sources and Properties. PDF-PSD
[5-7 marks]

b) Noise analysis: OpAmp – BJT
[12-15 marks]

c) Noise Figure.
[5-7 marks]

Invariants

- The mean-square value can be separated into a time-invariant part and a time-varying part.
- The time-invariant or static part is the square of the *signal average* or *mean value*,

$$\mu_x \equiv \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t)dt$$

- The time-varying or dynamic part of the mean-square value is the *signal variance*, which is denned as the mean-square value of *x(t)* about its mean value,

$$\sigma_x^2 \equiv \lim_{T\to\infty} \frac{1}{T} \int_0^T [x(t)-\mu_x]^2 dt$$

It follows that

$$\psi_x^2 = \mu_x^2 + \sigma_x^2$$

The positive square root of the variance is the *standard deviation*.

Probability Density Function

- The amplitude distribution of a random signal is described by the *probability density function* (PDF), *p(x),* defined as

$$p(x) \equiv \lim_{\Delta x \to 0} \frac{\text{Prob}\left[x < x(t) < x + \Delta x\right]}{\Delta x} = \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left[\lim_{T \to \infty} \frac{T_x}{T}\right]$$

- where *Tx* is the amount of time in which *x(t)* falls inside the amplitude interval from x to $x + \Delta x$.
- Therefore, the PDF gives the probability that the signal amplitude at any arbitrary moment lies inside a given amplitude range.

Power Spectral Density (PSD)

- The *power spectral density* (PSD) consider the distribution of noise power in different frequency bands.
- The PSD of a random signal *x(t)* is

$$G_{xx}(f) = \lim_{\Delta f \to 0} \frac{\psi_x^2(f, \Delta f)}{\Delta f} = \lim_{\Delta f \to 0} \frac{1}{\Delta f}\left[\lim_{T \to \infty} \frac{1}{T} x^2(t, f, \Delta f) dt\right]$$

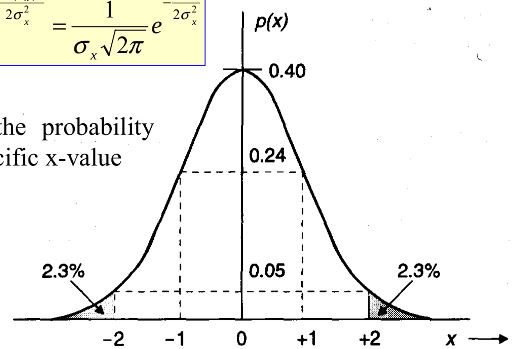
- where $\psi_x^2(f, \Delta f)$ is the signal power in the frequency band from f to $f+\Delta f$ and $x(t, f, \Delta f)$) is that part of $x(t)$ contributing to power in the frequency band from f to $f+\Delta f$.

Gaussian (normal) PDF

- Electronic noise has a Gaussian PDF because it results from a large number of random, independent events.
- This means that its PDF is bell-shaped and follows the equation:

$$p(x) = \frac{1}{\sigma_x\sqrt{2\pi}} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} = \frac{1}{\sigma_x\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma_x^2}}$$

Ordinates are the probability density of a specific x-value **(in σ–units !!!).**



PDF Tutorial

- The PDF function of the thermal noise is given by the following expression:

$$p(x) = \frac{1}{\sigma_x\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma_x^2}}$$

- Assuming a room temperature T = 300 K calculate:
- (i) rms value of noise voltage of 10 kΩ resistor over the bandwidth 10 MHz;
- (ii) the probability of 80 μV noise voltage.

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PDF Tutorial

- Calculate: rms value of noise voltage of 10 kΩ resistor over the bandwidth 10 MHz;
- The spectral density of the thermal noise is

$$\frac{\overline{v_n^2}}{\Delta f} = 4kT \cdot R = 4 \times 1.38 \cdot 10^{-23} \times 300 \times 10 \cdot 10^{3} =$$

$$= 1.656 \cdot 10^{-20} V^2 \times 10^4 = 1.656 \cdot 10^{-16} V^2 / Hz$$

- And the rms value is

$$v(rms) = \sqrt{\overline{v_n^2} \cdot \Delta f} = \sqrt{1.656 \cdot 10^{-16} \times 10 \cdot 10^{6}} =$$

$$= \sqrt{1.656 \cdot 10^{-9} V^2} = 4.07 \cdot 10^{-5} \mu V \approx 40 \mu V$$

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- The mean square value of noise (signal variance) is
- $\sigma_x \equiv v\ (rms) \approx 40 \mu V$
- In normalised coordinates $x = 80 \mu V = 2\sigma_x$
- Substituting x=2σ in PDF equation gives:

$$p(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp(-\frac{x^2}{2\sigma_x^2}) \approx 0.4 \exp(-\frac{4\sigma_x^2}{2\sigma_x^2})$$

$$= 0.4 \exp(-2) = 0.0533 \approx 5\%$$

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Signal to Noise ratio

- The common parameter to characterize signals is their power.
- In all practical systems the signal always coexists with noise.
- Therefore it can be described by very important parameter Signal-to-Noise Ratio (SNR or S/N):

$$SNR(dB) \equiv 10 \cdot \lg\left(\frac{Signal\ Power}{Noise\ Power}\right)$$

Noise Figure

- The ***noise figure (F)*** specifies the noise performance of a circuit or a device.
- The definition of the noise figure of a circuit is

$$F \equiv \frac{Input\ S/N\ Ratio}{Output\ S/N\ Ratio}$$

- Its disadvantage is that it is limited to situations where the source impedance is resistive
- However, it is widely used as a measure of noise performance in communication systems where the source impedance is often resistive.

Noise Figure (cont.)

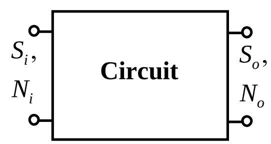
- Consider a circuit as shown below, where S represents signal power and N represents noise power.
- N_O is the total output noise including the circuit contribution and noise transmitted from the source resistance. The noise figure is
.

$$F = \frac{S_i}{N_i} \cdot \frac{N_o}{S_o}$$

Noticing that $S_o = GS_i$

$$F = \frac{N_o}{G \cdot N_i}$$

$$F = \frac{\textit{Total Noise Power}}{\textit{part of noise due to source resistance}}$$



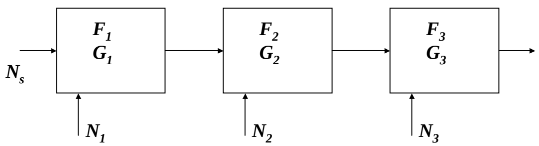
Noise Figure (P1)

- What is meant by the *Noise Figure* of an amplifier ?
- A three-stage amplifier has an input stage which-has a noise figure F_1 = 1 dB and a gain G_1 = 40dB. The two succeeding stages have respective noise figures F_2 = 3 dB and F_3 = 6 dB and corresponding gains G_2 = 20 dB and G_3 = 0 dB.
- Determine the overall noise figure of the amplifier.
- Which stage contributes most to overall noise?

Solution (P1)

$$Noise\ Figure \equiv \frac{Total\ Noise\ Power\ Output}{Output\ Noise\ Due\ to\ Rs} \equiv \frac{(S/N)_{in}}{(S/N)_{out}}$$

- The noise model for this 3-stage amplifier can described by the following diagram



$$NoiseFigure = \frac{G_1G_2G_3(N_S+N_1)+G_2G_3N_2+G_3N_3}{G_1G_2G_3N_S} =$$

$$= 1 + \frac{N_1}{N_S} + \frac{N_2}{G_1N_S} + \frac{N_3}{G_1G_2N_S} = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1G_2}$$

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Solution (P1)

- $F_1 = 1\ \text{dB} \equiv 1.259$
- $F_2 = 3\ \text{dB} \equiv 1.99$
- $F_3 = 6\ \text{dB} \equiv 3.98$
- $G_1 = 40\ \text{dB} \equiv 100$
- $G_2 = 20\ \text{dB} \equiv 10$
- $G_3 = 0\ \text{dB} \equiv 1$

$$F = 1 + (F_1 - 1) + (F_2 - 1)/G_1 + (F_3 - 1)/G_1G_2$$

$$F = 1.259 + (1.995 - 1)/100 + (3.98 - 1)/(100 \times 10) =$$
$$= 1.259 + 0.995/100 + 2.98/1000 =$$
$$= 1 + 0.259 + 0.00995 + 0.00298 = 1.272 \equiv 1.044dB$$

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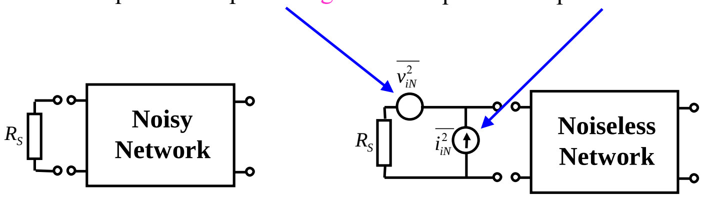
Equivalent input noise Generators

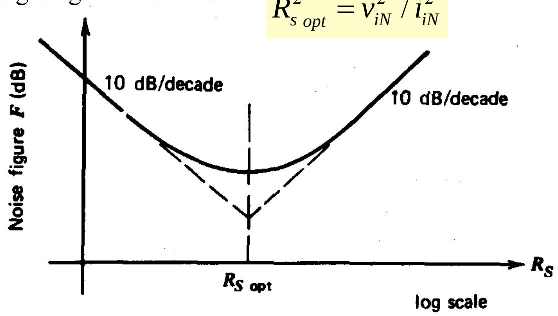
- The equivalent input noise voltage for a particular configuration is dependent on the source resistance R_S, as well as the transistor parameters.
- This method is now extended to a more general and more useful representation by using *two* equivalent input noise generators:
  - equivalent input voltage
  - equivalent input current



Noise Figure vs. Rs

- For very low values *of Rs*, the *v*-generator is dominant, whereas for large *Rs* the *i*-generator is the most important.
- It is apparent, that *F* has a minimum as *Rs* varies.
- By differentiating *F* with respect to *Rs*, we can calculate the value of *Rs* giving minimum *F*:

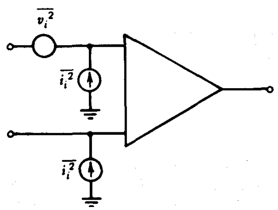
$$R_{s\ opt}^2 = \overline{v_{iN}^2} / \overline{i_{iN}^2}$$



Noise model for 741

- Noise model for OpAmp circuits is presented below,
- The concrete form for input noise generators depends on particular circuit design

For 741 OpAmp these expressions are:

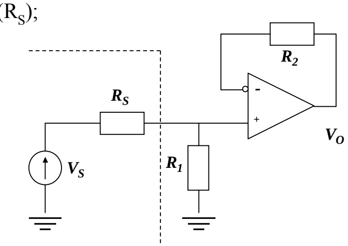


$$\frac{\overline{v_i^2}}{B} = 4kT(16000)$$

$$\frac{\overline{i_i^2}}{B} = 2qI_B \approx 2q(0.2\cdot 10^{-6})$$

(P2) OpAmp Noise

- The amplifier circuit shown in Fig. 1 has an effective noise bandwidth of 5 MHz. Given that the differential amplifier has equivalent input noise current and voltage densities of 1 pA/√Hz and 1 nV/√Hz respectively
- sketch the noise equivalent circuit and hence determine values for:
- (a) The equivalent input noise current and voltage of the circuit (excluding the source);
- *(b)* The output signal-to-noise (S/N) ratio, given a 1 V (rms) sinusoidal source (V_S) and a 10 kΩ source resistance (R_S);
- (c) The noise figure (F)
- of the amplifier circuit.
- Assume that $4kT$= 1.65 x 10^{-20} J.

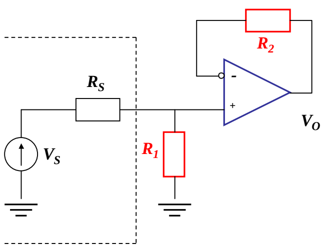


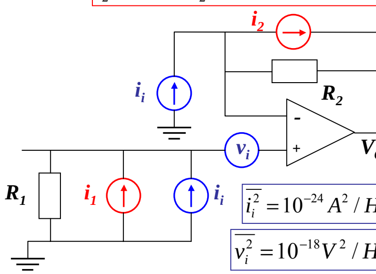
Solution (P2a)

- The noise equivalent circuit consists of two equivalent input noise currents i_i and one voltage v_i and
- two thermal noises (spectral densities) from R_1 and R_2 - i_1 and i_2

$$\overline{i_1^2} = 4kT / R_1 = 1.65\times10^{-25}\ \text{A}^2/\text{Hz}$$

$$\overline{i_2^2} = 4kT / R_2 = 1.65\times10^{-24}\ \text{A}^2/\text{Hz}$$

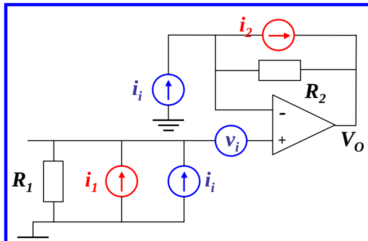


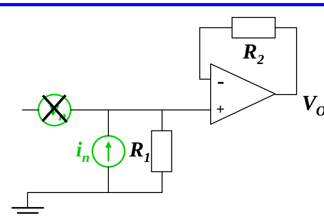


$$\overline{i_i^2} = 10^{-24} A^2 / Hz$$

$$\overline{v_i^2} = 10^{-18} V^2 / Hz$$

Solution (P2a)





For open-circuit inputs:

The output of the circuit in Fig.2 is $\overline{v_o^2} = \left(\overline{i_1^2} + \overline{i_i^2}\right)R_1^2 + \overline{v_i^2} + \left(\overline{i_i^2} + \overline{i_2^2}\right)R_2^2$ (1)

The output of the circuit in Fig.3 is $\overline{v_o^2} = \overline{i_n^2} R_1^2$ (2)

Equating (1) and (2) gives the expression for the equivalent input noise current

$$\overline{i_n^2} = \left(\overline{i_1^2} + \overline{i_i^2}\right) + \overline{v_i^2} / R_1^2 + \left(\overline{i_i^2} + \overline{i_2^2}\right) R_2^2 / R_1^2$$

Solution (P2a)

Hence

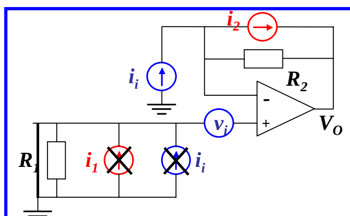
$$\overline{i_n^2} = \left(\overline{i_1^2} + \overline{i_i^2}\right) + \overline{v_i^2} / R_1^2 + \left(\overline{i_i^2} + \overline{i_2^2}\right) R_2^2 / R_1^2 =$$

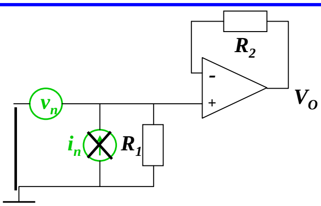
$$= (10^{-24} + 0.165 \cdot 10^{-24}) + 10^{-18} / 10^{10} + (10^{-24} + 1.65 \cdot 10^{-24}) / 10^2$$

$$= 1.165 \cdot 10^{-24} + 10^{-28} + 1.165 \cdot 10^{-26} = 1.182 \cdot 10^{-24} \, A^2 / Hz$$

$$i_n = \sqrt{\overline{i_n^2} \times \Delta f} = \sqrt{1.182 \cdot 10^{-24} \times 5 \cdot 10^6} = 2.43 \, nA$$

Solution (P2a)





For short-circuit inputs:

The output of the circuit in Fig.2 is $\overline{v_o^2} = \overline{v_i^2} + \left(\overline{i_i^2} + \overline{i_2^2}\right) R_2^2$ (3)

The output of the circuit in Fig.3 is $\overline{v_o^2} = \overline{v_n^2}$ (4)

Equating (3) and (4) gives the expression for the equivalent input noise voltage

$$\overline{v_n^2} = \overline{v_i^2} + \left(\overline{i_i^2} + \overline{i_2^2}\right) R_2^2 = 10^{-18} + \left(10^{-24} + 1.65 \cdot 10^{-24}\right) \times 10^8 \approx 2.65 \cdot 10^{-16} V^2 / Hz$$

$$v_n = \sqrt{\overline{v_n^2} \times \Delta f} = \sqrt{2.65 \cdot 10^{-16} \times 5 \cdot 10^6} = 35.8 \, \mu V$$

Solution (P2b)

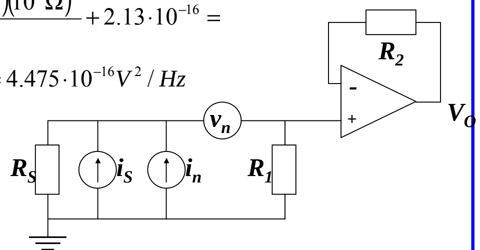
(b) determine the output S/N, $V_S = 1$ V (rms) and $R_S = 10$ kΩ

$$\overline{v_{on}^2} = \frac{\left(\overline{i_n^2}+\overline{i_S^2}\right)\left(R_1\times R_S\right)^2}{\left(R_1+R_S\right)^2} + \frac{\overline{v_n^2}R_1^2}{\left(R_1+R_S\right)^2} =$$

$$= \frac{\left(1.182\cdot10^{-24}+4kT/10k\Omega\right)\left(100k\Omega\times10k\Omega\right)^2}{\left(100k\Omega+10k\Omega\right)^2} + \frac{2.575\cdot10^{-16}\times\left(100k\Omega\right)^2}{\left(100k\Omega+10k\Omega\right)^2} =$$

$$= \frac{\left(1.182\cdot10^{-24}+1.65\cdot10^{-24}\right)\left(10^9\Omega\right)^2}{\left(1.1\cdot10^5\Omega\right)^2} + 2.13\cdot10^{-16} =$$

$$= 2.35\cdot10^{-16}+2.13\cdot10^{-16} = 4.475\cdot10^{-16}V^2/Hz$$



Solution (P2b,c)

$$v_{on} = \sqrt{\overline{v_{on}^2}\times\Delta f} = \sqrt{4.475\cdot10^{-16}\times5\cdot10^6} = 47\ \mu V$$

$$V_{oS} = \frac{R_1}{R_1+R_S}\times1V = \frac{100k\Omega}{100k\Omega+10k\Omega}\times1V \approx 0.9V$$

S/N_{out} = 0.9 V / 47 μV = 19148 ≡ 85.64 dB

(c) (determine) The noise figure (F) of the amplifier circuit

$$F \equiv \frac{S/N_{in}}{S/N_{out}}$$

$$S/N_{in} = \frac{v_S}{v_{Sn}} = \frac{v_S}{\sqrt{4kT/R_S\times\Delta f}} = 35749 \equiv 91dB$$

$$F = \frac{S/N_{in}}{S/N_{out}} = \frac{35749}{19148} = 1.84$$

$$F(dB) = 10\log(F) = 10\log(1.84) = 2.66dB$$

Solution (P2 additional)

(d) Determine the optimal value of R_S for amplifier circuit

Solution: the optimal value of R_S is

$$R_{Sopt} = v_n / i_n = 35\mu V / 2.43nA = 14.7k\Omega$$

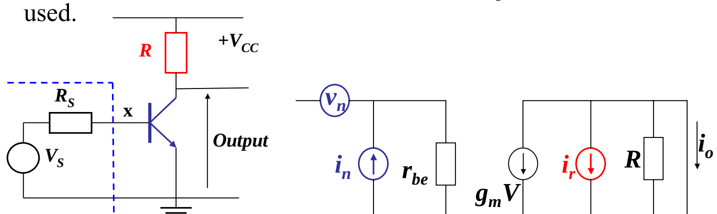
(e) Determine the noise figure (F) of the circuit with optimal R_S

Solution: Repeat (c)-(d)

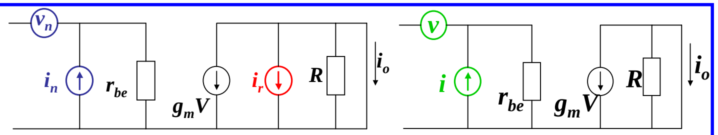
$$F_{opt} = 1.79 \qquad F_{opt}(dB) = 2.54dB$$

(P3) BJT Noise

- The transistor used in the common-emitter circuit shown in Fig.3 has an equivalent input noise spectral density values of 1 nV/√Hz and 1 pA/√Hz and device a.c. parameter values g_m = 100 mA/V and r_{be} = 1 kΩ. The load resistance value R = 10 kΩ. Bias circuitry has been omitted. Determine:
- (i) The equivalent input noise voltage and current spectral densities at the amplifier input (X).
- (ii) The optimum value of source resistance R_s which should be used.



Solution (P3a)



- For open circuit inputs $$i_o^2 = g_m^2 r_{be}^2 i_n^2 + i_r^2 = g_m^2 r_{be}^2 i^2$$

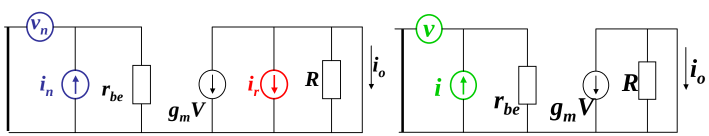
$$i^2 = i_n^2 + \frac{i_r^2}{g_m^2 r_{be}^2} = (1\,pA/\sqrt{Hz})^2 + \frac{4kT/R}{g_m^2 r_{be}^2} =$$

$$= 10^{-24}\,A^2/Hz + \frac{4 \times 1.38 \text{ x } 10^{-23}\text{ J/K} \cdot 298\text{ K}/10^4\,\Omega}{(0.1A/V \times 1000\Omega)^2} =$$

$$= 10^{-24}\,A^2/Hz + \frac{1.65 \text{ x } 10^{-24}\;A^2/Hz}{10^4} =$$

$$10^{-24}\,A^2/Hz + 1.65 \text{ x } 10^{-28}\;A^2/Hz \approx 10^{-24}\,A^2/Hz$$

Solution (P3a,b)



- For short circuit inputs $$i_o^2 = g_m^2 v_n^2 + i_r^2 = g_m^2 v^2$$

$$v^2 = v_n^2 + i_r^2/g_m^2 = 10^{-18} V^2/Hz + \frac{1.65 \cdot 10^{-24} A^2/Hz}{10^{-2} A^2/V^2} =$$

$$= 10^{-18} V^2/Hz + 1.65 \cdot 10^{-22} V^2/Hz = 1.000165 \cdot 10^{-18} V^2/Hz$$

(b) The optimum Rs is

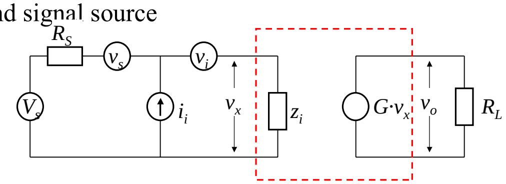
$$R_S = \sqrt{v^2/i^2} = \sqrt{10^{-18} V^2/\sqrt{Hz}/10^{-24} A^2/\sqrt{Hz}} = \sqrt{10^6\,\Omega^2} = 1\,k\Omega$$

Problem 4

- The voltage source of 100mV with resistance 1 kΩ is connected to the amplifier with 10 kΩ input resistance and 20 dB gain. The amplifier has an equivalent input noise spectral density values of 1 μV/√Hz and 1 nA/√Hz. The load resistance value R = 100 kΩ and frequency bandwidth B =1 MHz. Determine:
- (a) The noise figure (F)
- (b) The output signal-to-noise (S/N) ratio;
- (c) The optimum value of source resistance R_s .
- (d) The optimised noise figure and output signal-to-noise ratio.
- Assume that 4kT = 1.656 x 10^{-20} J

P4 Solution (cont.)

- Consider the circuit below, consisting of the noise generators and signal source



- The noise at input terminals from the noise generators is

$$v_{xA} = v_i \frac{z_i}{z_i + R_S} + i_i \frac{z_i R_S}{z_i + R_S}$$

and thus

$$\overline{v_{xA}^2} = \overline{v_i^2} \frac{|z_i|^2}{|z_i + R_S|^2} + \overline{i_i^2} \frac{|z_i R_S|^2}{|z_i + R_S|^2}$$

P4 Solution (cont.)

The noise power in R_L from noise generators is

$$N_{oA} = \frac{|G|^2}{R_L}\overline{v_{xA}^2} = \frac{|G|^2}{R_L}\left[\overline{v_i^2}\frac{|z_i|^2}{|z_i+R_S|^2} + \overline{i_i^2}\frac{|z_i R_S|^2}{|z_i+R_S|^2}\right]$$

The noise power in R_L from source resistor is

$$N_{oB} = \frac{|G|^2}{R_L}\frac{|z_i|^2}{|z_i+R_S|^2}\overline{v_s^2} = \frac{|G|^2}{R_L}\frac{|z_i|^2}{|z_i+R_S|^2}\cdot 4kTR_sB$$

Finally, the Noise Figure is

$$F = \frac{N_{oA}+N_{oB}}{N_{oB}} = 1 + \frac{N_{oA}}{N_{oB}} = 1 + \frac{\frac{|G|^2}{R_L}\left[\overline{v_i^2}\frac{|z_i|^2}{|z_i+R_S|^2} + \overline{i_i^2}\frac{|z_i|^2 R_s^2}{|z_i+R_S|^2}\right]}{\frac{|G|^2}{R_L}\frac{|z_i|^2}{|z_i+R_S|^2}4kTR_sB} =$$

$$= 1 + \frac{\left[\overline{v_i^2} + \overline{i_i^2}R_s^2\right]}{4kTR_sB} = 1 + \frac{\left[\overline{v_i^2} + \overline{i_i^2}R_s^2\right]}{4kTR_sB} = 1 + \frac{\overline{v_i^2}}{4kTR_sB} + \frac{\overline{i_i^2}}{4kTB/R_s}$$

P4 Solution (cont.)

Finally, the Noise Figure is

$$F = 1 + \frac{\overline{v_i^2}}{4kTBR_s} + \frac{\overline{i_i^2}R_s}{4kTB} = 1 + \frac{10^{-12}V^2/Hz}{1.656\cdot10^{-20}J\times10^6Hz\times10^3\Omega} + \frac{10^{-18}A^2/Hz\times10^3\Omega}{1.656\cdot10^{-20}J\times10^6Hz} =$$

$$= 1 + \frac{10^{-12}}{1.656\cdot10^{-11}} + \frac{10^{-15}}{1.656\cdot10^{-14}} = 1 + \frac{10^{-1}}{1.656} + \frac{10^{-1}}{1.656} = 1 + 0.06 + 0.06 = 1.12$$

w

P4 Solution (cont.)

The overall input noise is

$$v_{xn} = \sqrt{(\overline{v_{xA}^2} + \overline{v_{xB}^2}) \cdot B}$$

The overall output noise is

$$v_{on} = |G| v_{xn} = |G| \sqrt{\left[\overline{v_i^2} \frac{|z_i|^2}{|z_i + R_S|^2} + \overline{i_i^2} R_s^2 \frac{|z_i|^2}{|z_i + R_S|^2} + \frac{|z_i|^2}{|z_i + R_S|^2} 4kTR_s \right] \cdot B} =$$

$$= \frac{|G| \cdot |z_i|}{|z_i + R_S|} \sqrt{\left[\overline{v_i^2} + \overline{i_i^2} R_s^2 + 4kTR_s \right] \cdot B} =$$

$$\frac{10 \cdot 10k\Omega}{|10k\Omega + 1k\Omega|} \sqrt{\left[10^{-12} + 10^{-18} \cdot 10^6 + 1.656 \cdot 10^{-20} J \cdot 10^3 k\Omega\right] \cdot 10^6 Hz} =$$

$$= 9.09\sqrt{2 \cdot 10^{-6} + 1.656 \cdot 10^{-11}} V = 12.85mV$$

P4 Solution (cont.)

The output signal is

$$V_{So} = |G| \cdot \frac{|z_i|^2}{|z_i + R_S|^2} V_S = 10 \frac{10k\Omega}{11k\Omega} 100mV = 909mV$$

The output signal –to-noise ratio is

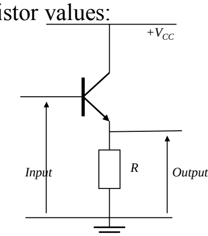
$$S / N = V_S / v_{on} = 909mV / 12.85mV = 70.74 = 37dB$$

The optimal source resistor R_S is

$$R_{Sopt} = v_i / i_i = 1\mu V / 1nA = 10^{-6} / 10^{-9} \Omega = 1k\Omega$$

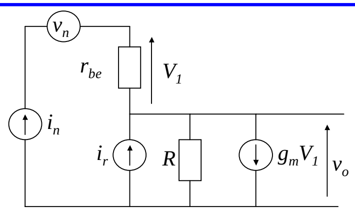
(P5) EF BJT Noise

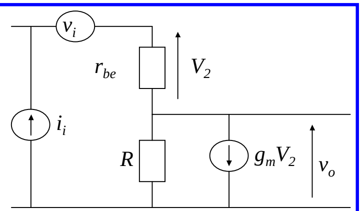
- The BJT device contained in the circuit shown in Fig. 1 has an equivalent input noise voltage of e_n=1 nV/√Hz and an equivalent input noise current of i_n = 1 pA/√Hz at a particular spot frequency. Given the following relevant transistor parameter and resistor values:
- r_{be} = 1 kΩ; g_m = 0.1 A/V; R = 1 kΩ



- determine the corresponding equivalent input noise voltage and current values (I/Hz) for the circuit and the optimum value of source resistance required. Assume that 4kT = 1.656 x 10^{-20}.

(P5) EF BJT Noise





- For short circuit inputs: $\overline{V_o} = R\left[\overline{i_r} + \overline{V_1}(g_m + 1/r_{be})\right] = \overline{i_r}R + \frac{R(g_m + 1/r_{be})(\overline{v_n} + \overline{i_r}R)}{1 + Rg_m + R/r_{be}}$

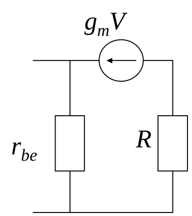
$$\overline{V_1} = \frac{\overline{v_n} + \overline{i_r}R}{1 + g_m R + R/r_{be}} \qquad \overline{V_o} = R\left[g_m + 1/r_{be}\right]\overline{V_2} = \frac{R\left[g_m + 1/r_{be}\right]\overline{V_i}}{1 + R\left[g_m + 1/r_{be}\right]}$$

$$\overline{V_i} = \overline{V_2} + R\left[g_m + 1/r_{be}\right]\overline{V_2} \qquad \overline{V_2} = \frac{\overline{V_i}}{1 + R\left[g_m + 1/r_{be}\right]}$$

$$\overline{v_n} = \overline{V_1} + R\left[\overline{i_r} + g_m\overline{V_1} + \overline{V_1}/r_{be}\right] = \overline{V_1} + R\left[\overline{i_r} + \overline{V_1}(g_m + 1/r_{be})\right]$$

(P6) CB BJT Noise

- A common-base amplifier with a load resistance of 1 kΩ and operating at a quiescent (collector) current of 2.5 mA has parameter values of g_m = 100 mA/V and r_{be} = 2 kΩ.
- It is suggested that a reduction in quiescent current would allow the input impedance of the amplifier to match that of a 500 signal source. To what value should the quiescent current be lowered ?
- Solution
- Input impedance $Z_{in} = \frac{r_{be}}{1+g_m r_{be}} \approx \frac{1}{g_m}$
- At I_C=2.5 mA, $g_m = 0.1$ A/V , $Z_{in} = 10\ \Omega$
- At I_C=0.5 mA, $g_m = 0.02$ A/V, $Z_{in} = 50\ \Omega$



(P6) CB BJT Noise

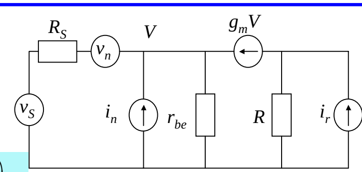
- Determine the (mid-band) power spectral density of the noise observed at the output of the amplifier in the case of the revised quiescent current value given that the transistor has equivalent input current and voltage noise source values of 1 pA/√Hz and 1 nV/√Hz respectively.
- Is the reduction in quiescent current likely to improve or disimprove noise performance ?
- Assume that 4kT = 1.656 x $10^{'20}$ J.

(P6) CB BJT Noise

$$V^2 = \left[\frac{V_n^2+V_S^2}{R_S^2}+i_n^2\right]\left[\frac{r_{be}R_S}{r_{be}+R_S}\right]^2$$

$$V_O^2 = \left(g_m^2 V^2 + i_R^2\right)R^2$$

$$V_O^2 = \left(\left[\frac{V_n^2+V_S^2}{R_S^2}+i_n^2\right]\left[\frac{r_{be}R_S g_m}{r_{be}+R_S}\right]^2 + i_R^2\right)R^2$$



$$g_m = 0.02;\ r_{be} = 10k\Omega;\ R_S = 50\Omega;\ V_n^2 = (1nV)^2;\ i_n^2 = (1pA)^2;$$

$$V_S^2 = 4kTR_S;\ i_R^2 = 4kT/R_S;\ g_m r_{be} = 200;$$

$$V_O^2 = \left(\left[\frac{10^{-18}+1.656\cdot 10^{-20}\times 50}{2500}+10^{-24}\right]\left[\frac{200\times 50}{10k\Omega+50\Omega}\right]^2 + 1.656\cdot 10^{-20}/50\right)\cdot 10^6 =$$

$$= \left(\left[\frac{10^{-18}+0.828\cdot 10^{-18}}{2500}+10^{-24}\right]\times 0.99 + 3.31\cdot 10^{-18}\right)\cdot 10^6 =$$

$$= \left(7.322\cdot 10^{-22}\times 0.99 + 1.656\cdot 10^{-23}\right)\cdot 10^6 \approx 7.41\cdot 10^{-16} V^2/Hz$$

(P6) CB BJT Noise

- v_o = 27.23 nV/√Hz,
- *Note if* $g_m \downarrow$, then $r_{be} \uparrow$
- Reduction in quiescent current will increase r_{be}, i.e. the noise performance should be improved

$$V_O^2 = \left(\left[\frac{V_n^2+V_S^2}{R_S^2}+i_n^2\right]\left[\frac{r_{be}R_S g_m}{r_{be}+R_S}\right]^2 + i_R^2\right)R^2$$